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EFFICIENT SCHEME FOR ENTANGLED STATES WITH TRAPPED ATOMS IN A RESONATOR

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A protocol is proposed to generate atomic entangled states in a cavity QED system. It utilizes Raman transitions or stimulated Raman adiabatic passages between two systems to entangle the ground states of two three-state Λ -type atoms trapped in a single mode cavity. It does not need the measurements on cavity field nor atomic detection and can be implemented in a deterministic fashion. Since the present protocol is insensitive to both cavity decay and atomic spontaneous emission, the produced entangled states may have some interesting applications in quantum information processing.

Keywords: Atomic entangled state; Cavity QED; Quantum information processing.

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1. Introduction

Entangled states not only could be utilized to test fundamental quantum mechanical principles such as Bell's inequalities¹ but also play a central role in practical applications of quantum information processing², such as quantum computation^{3,4}, quantum teleportation⁵, and quantum cryptography⁶. It is generally believed that atoms are good candidates for storing quantum information and are natural quantum information processors. Therefore, producing atomic entangled states are particularly significant. In the context of cavity QED^{7,8}, numerous proposals have been presented for generating atomic entangled states^{9,10,11,12,13,14,15,16,17,18,19,20}. Though these proposals seem very promising, some rely on measurements on photons, which impairs their performance; some are not immune to atomic spontaneous emission.

In this paper, we propose a protocol for the realization of atomic entangled states with a cavity QED system. It consists of two three-state Λ -type atoms and a single mode cavity. We show that, through suitably choosing the detunings and intensities of fields, Raman transitions or stimulated Raman adiabatic passages (STIRAP)²¹ between the two atoms can be achieved, which can be utilized to produce the atomic entangled states. This proposal could be implemented in a deterministic fashion, thus requiring no measurements on cavity field and atomic states. Because the atomic excited states and cavity mode excitations are not involved in this process,

this protocol is very robust against atomic spontaneous emission and cavity photon decay. Since the STIRAP techniques²¹ are utilized, the protocol is very robust against moderate fluctuations of experimental parameters. With presently available experimental setups in cavity QED, this proposal could be implemented.

2. Generation of atomic entangled states through Raman transitions

Consider the case of two three-state Λ -type atoms trapped in a single mode cavity. As sketched in Fig. 1, each atom has the level structure of a Λ system with two stable ground states $|0\rangle$ and $|1\rangle$, and an excited state $|e\rangle$. The classical field of frequency ω_L drives dispersively the transition $|0\rangle \leftrightarrow |e\rangle$ with the Rabi frequency Ω_i ($i = 1, 2$) and detuning $\Delta = \omega_{e1} - \omega_L$. The cavity mode of frequency ν couples the transition $|1\rangle \leftrightarrow |e\rangle$ with the coupling constant g_i and the same detuning $\Delta = \omega_{e0} - \nu$. For simplicity the coupling constants of both atoms to the cavity mode are taken to be the same, $g_1 = g_2 = g$, but this is not the necessary condition for the analysis. In addition, we neglect the position dependence of the cavity-atom coupling strengths by assuming the Lamb-Dicke limit. In the interaction picture, the associated Hamiltonian under the dipole and rotating wave approximation is given by (let $\hbar = 1$)

$$\hat{H}_I = \sum_{i=1,2} (\Omega_i \hat{\sigma}_{e0}^i e^{i\Delta t} + g_i \hat{a} \hat{\sigma}_{e1}^i e^{i\Delta t}) + \text{H.c.}, \quad (1)$$

where $\hat{\sigma}_{jm} = |j\rangle\langle m|$ is the atomic transition operator, and \hat{a} is the annihilation operator for the cavity mode. We consider dispersive detuning $|\Delta| \gg |\Omega_i|, |g|$ for each atom. Since level $|e\rangle$ is coupled dispersively with both levels $|0\rangle$ and $|1\rangle$, it can be adiabatically eliminated and atomic spontaneous emission can be neglected²². Then we obtain the effective Hamiltonian describing the Raman excitations of the atoms

$$\begin{aligned} \hat{H}_{\text{eff}} = & \sum_{i=1,2} \left(\frac{|g|^2}{\Delta} \hat{a}^\dagger \hat{a} \hat{\sigma}_{11}^i + \frac{|g|^2 m}{\Delta} \hat{\sigma}_{00}^i \right. \\ & \left. + \frac{|\Omega_i g^*|}{\Delta} \hat{a}^\dagger \hat{\sigma}_{10}^i + \frac{|\Omega_i^* g|}{\Delta} \hat{a} \hat{\sigma}_{01}^i \right), \end{aligned} \quad (2)$$

where we chose Ω_i in phase with g . We have included an energy shift $\Delta_m = \frac{|g|^2 m - |\Omega_i|^2}{\Delta}$ ($m = 0, 1, \dots$) to level $|0\rangle$ for each atom, which could be implemented through the action of external classical fields. The number m is introduced for convenience, which can be determined from the expression for the energy shift Δ_m , and can be controlled through tuning the external classical fields. For instance, if we tune the external classical fields such that the energy shift has the value $-\frac{|\Omega_i|^2}{\Delta}$, this corresponds to choosing $m = 0$. In the same way, we can choose m for other values.

Assume that atoms 1 and 2 are initially prepared in their stable ground states $|0\rangle_1$ and $|1\rangle_2$, and the cavity field is in vacuum state $|0\rangle_C$. Then the dynamics is

confined to the subspace of the collective energy levels of the two atoms and cavity mode $\{|01;0\rangle, |11;1\rangle, |10;0\rangle\}$, where $|ij;k\rangle = |i\rangle_1|j\rangle_2|k\rangle_C$ ($i, j, k = 0, 1$) describes a system with the atoms in state $|i\rangle_1|j\rangle_2$ and cavity in Fock state $|k\rangle_C$. Then we can write the Hamiltonian of Eq. (2) in this subspace as

$$\begin{aligned}\hat{H}_{\text{eff}} = & \frac{m|g|^2}{\Delta}(|10;0\rangle\langle 0;01| + |01;0\rangle\langle 0;10|) + \frac{2|g|^2}{\Delta}|11;1\rangle\langle 1;11| \\ & + \frac{|g\Omega_1^*|}{\Delta}|01;0\rangle\langle 1;11| + \frac{|g\Omega_2^*|}{\Delta}|10;0\rangle\langle 1;11| + \text{H.c.},\end{aligned}\quad (3)$$

which forms a typical Λ system. If we assume that $m = 0$ and $\frac{2|g|^2}{\Delta} \gg \{\frac{|g\Omega_1^*|}{\Delta}, \frac{|g\Omega_2^*|}{\Delta}\}$, then we obtain a Raman transition between states $|01;0\rangle$ and $|10;0\rangle$. In this case, through adiabatic elimination of the state $|11;1\rangle$, we get an effective Hamiltonian describing the Raman excitation

$$\hat{H}_{\text{eff}} = \Theta|10;0\rangle\langle 0;10| + \text{H.c.}, \quad (4)$$

with $\Theta = \frac{|\Omega_1\Omega_2|}{2\Delta}$ being the Raman transition rate. The Hamiltonian (4) describes a two photon Raman transition between two distant atoms trapped in a cavity.

The system is initially prepared in the state $\psi(0) = |01;0\rangle$, then the state evolution of the system is given by

$$\psi(t) = \cos(\Theta t)|01;0\rangle - i \sin(\Theta t)|10;0\rangle, \quad (5)$$

which is an entangled state for the two atoms. If we choose $\Theta t = \pi/4$, we could obtain the maximally entangled two-atom state

$$\psi_a = \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle), \quad (6)$$

which is the well-known EPR state. This entangled state is very robust because it only involves the ground states of the atoms.

3. Atomic entanglement through stimulated Raman adiabatic passages

Although the entanglement mechanism given above could work well for a pair of atoms, it relies on off-resonance Raman transitions and is not robust enough. We now extend the idea to the case of on-resonance STIRAP process between two trapped atoms in a cavity. We will see that this STIRAP protocol is more robust than the Raman excitation based scheme. Different from the general STIRAP process²¹, the present transfer is between ground states of two atoms and keeps cavity field from exciting.

Assume that the two trapped atoms are far apart so that they can be addressed individually by laser beams with time dependent Rabi frequencies. We also suppose that the amplitudes of $\Omega_1(t), \Omega_2(t)$ satisfy $\Omega_1 = -\xi\Omega_2$ in the paper, where ξ is a

control parameter. If we choose $m = 2$, then from Eq. (3) we can get the following Hamiltonian

$$\hat{H}_{\text{eff}} = \frac{|g\Omega_1(t)|}{\Delta}|01;0\rangle\langle 1;11| - \frac{|g\Omega_2(t)|}{\Delta}|10;0\rangle\langle 1;11| + \text{H.c.}, \quad (7)$$

where we have discarded the constant energy terms. The effective Hamiltonian (7) describes a typical Λ system which is on resonance. Therefore, dark state exists in this system,

$$|D(t)\rangle = \cos\theta(t)|01;0\rangle + \sin\theta(t)|10;0\rangle, \quad (8)$$

with $\tan\theta(t) = |\Omega_1(t)|/|\Omega_2(t)|$. As a consequence, adiabatic transfer of population can occur between states $|01;0\rangle$ and $|10;0\rangle$ by slowly varying the laser amplitudes $\Omega_1(t)$ and $\Omega_2(t)$. This procedure resembles the STIRAP process, which transfers population between ground states of one atom, but the present transfer is between ground states of two atoms and keeps cavity field excitation from being involved in this process. A maximally entangled state can be generated in the particular case $|\Omega_1(t)| = |\Omega_2(t)|$, i.e., $1/\sqrt{2}(|01\rangle + |10\rangle)$. The generated entangled two-atom state is more robust than previously proposed entangled atomic states, due to the fact that the atomic excited states and cavity mode are unpopulated during the process.

To verify the above approximations and STIRAP process, we numerically simulate the dynamics generated by the full Hamiltonian (including terms describing cavity decay and atomic spontaneous emission) and compare it with the results generated by the effective model (7). In Fig. 2 the numerical results of the system evolution with decay terms for the atoms ($\Gamma = 0.1g$) and cavity modes ($\kappa = 0.1g$) are displayed. The Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$ are assumed to be Gaussian envelopes for the simulations, i.e., $\Omega_i(t) = \Omega_i e^{-(t-\tau_i)^2/\delta\tau_i^2}$ ($i = 1, 2$). Clearly, this process is an adiabatic passage of dark state (8), since the excited state $|e\rangle$ of each atom and photon states are vanishingly populated (less than 10^{-3}). Thus the numerical simulations clearly verify the analytical results.

In order to quantify the robustness of this protocol against cavity decay, atomic spontaneous emission, and fluctuations of experimental parameters, we evaluate the success rate P and fidelity F , and compare them with those obtained in other setups^{18,20}. Following the standard quantum theory of damping, we investigate the combined influence of the cavity decay and atomic spontaneous emission on the coupled system. After tracing out the reservoir degrees of freedom, we obtain the master equation for the density matrix of the atom-cavity system

$$\begin{aligned} \dot{\rho} = & -i[\hat{H}_I, \rho] + \kappa(2\hat{a}\rho\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\rho - \rho\hat{a}^\dagger\hat{a}) \\ & + \sum_{i=1,2;j=0,1} \frac{\gamma}{2}(2\hat{\sigma}_{je}^i\rho\hat{\sigma}_{ej}^i - \hat{\sigma}_{ee}^i\rho - \rho\hat{\sigma}_{ee}^i). \end{aligned} \quad (9)$$

The success rate P is defined as the probability of producing the entangled state $|\psi\rangle = 1/\sqrt{2}(|01\rangle + |10\rangle)$, and the fidelity $F = \langle\psi|\rho_a|\psi\rangle$, where $\rho_a = \text{Tr}_{\text{cavity}}(\rho)$

is the final reduced density matrix of the atoms. Fig. 3(a) shows the success rate and fidelity vs. $\kappa\gamma/g^2$ for this scheme and the setup proposed in Ref.¹⁸. We see that within the relatively strong coupling regime, the success rate is always close to unity for this protocol, while that for the setup proposed in Ref.¹⁸ is very sensitive to cavity decay. In addition, the fidelity for the maximally entangled state is about 99.9% in our proposal, but the fidelity in Ref.¹⁸ is just 93.5%. This is due to the fact that the present proposal does not involve cavity field excitation and is deterministic, requiring no measurement on the cavity field. In Fig. 3(b), we plot the fidelity for the entangled states prepared in the present scheme and in Ref.²⁰ vs. the fluctuation of the Rabi frequency $\delta\Omega/\Omega$. Here $\Omega = \max\{\Omega_1, \Omega_2\}$. We see that under relatively small fluctuations of the Rabi frequency, the fidelity is still very high for our protocol ($\geq 90\%$), but it may become very small for the scheme in Ref.²⁰. The scheme in Ref.²⁰ relies on fractional STIRAP techniques, which requires a precise ratio of pulse endings, thus impairing its performance if the intensities of the two classical lasers have small variations.

For experimental implementation of the proposal, one could utilize the recently performed experimental setups²³. There, trapped Cesium atoms could couple to a high-finesse Fabry-Perot cavity. Cs atoms are dropped from a magneto-optical trap into the cavity and cooled into a far off-resonant trap by an optical lattice. The states used in the setup are ground $|6S_{1/2}, F = 3, 4\rangle$ and excited $|6P_{3/2}, F = 3'\rangle$ manifolds. The experimental parameters are $g_1 \sim g_2 \sim g/(2\pi) = 16$ MHz, $(\Gamma, \gamma)/2\pi = (3.8, 2.6)$ MHz, and we choose $\Delta = 10g$, $\Omega_1 \sim 100$ MHz, $\Omega_2 \sim 100$ MHz, $\tau_1 \sim 3\mu s$, $\tau_2 \sim 1.5\mu s$, and $\delta\tau_1 \sim 1.3\mu s$, $\delta\tau_2 \sim 1.8\mu s$. With these parameters we obtain the fidelity up to 1 for entanglement with a total preparation time $t \sim 2\mu s$. The life time of the entangled state generated by the scheme is estimated to be about $20\mu s$. The effective life time of the photons in the experiment is about $1/(0.001\kappa) \sim 60\mu s$. Thus the present protocol could be implemented with these setups. Other promising devices are superconducting circuit devices²⁴, where superconducting qubits can be individually addressed using lasers in the transmission-line resonators.

4. Summary

To conclude, we have presented a protocol for the generation of atomic entangled states in a cavity QED system. It is based on Raman transitions or adiabatic Raman passages between two systems to entangle the ground states of two three-state Λ -type atoms trapped in a single mode cavity. This scheme needs neither the measurements on cavity field nor atomic detection. Because the atomic excited states and cavity field excitations are never involved in this proposal, our scheme is insensitive to both cavity decay and atomic spontaneous emission. Due to the STIRAP techniques, this proposal is robust against fluctuations of experimental parameters. Experimentally this protocol could be realized with the presently available technology in cavity QED.

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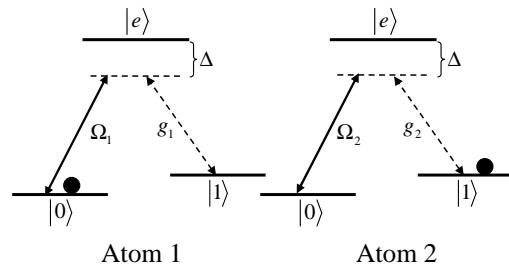


Fig. 1. Atomic levels of two atoms trapped in a single mode cavity.

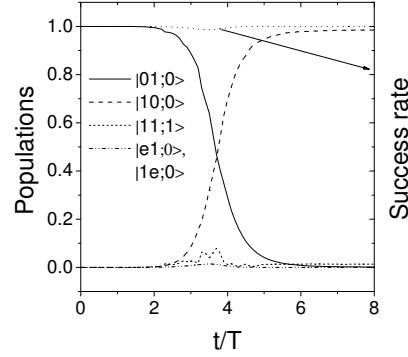


Fig. 2. Populations and success rate versus time from the full Hamiltonian for $\Gamma = 0.1g$, and $\kappa = 0.1g$, with $T = 50g^{-1}$. The parameters for the simulations are chosen as $|\Omega_1| = 2g$, $|\Omega_2| = g$, $\tau_1 = 0.3 \times 10^3 g^{-1}$, $\tau_2 = 0.15 \times 10^3 g^{-1}$, $\delta\tau_1 = 0.125 \times 10^3 g^{-1}$, $\delta\tau_2 = 0.175 \times 10^3 g^{-1}$, and $\Delta = 20g$.

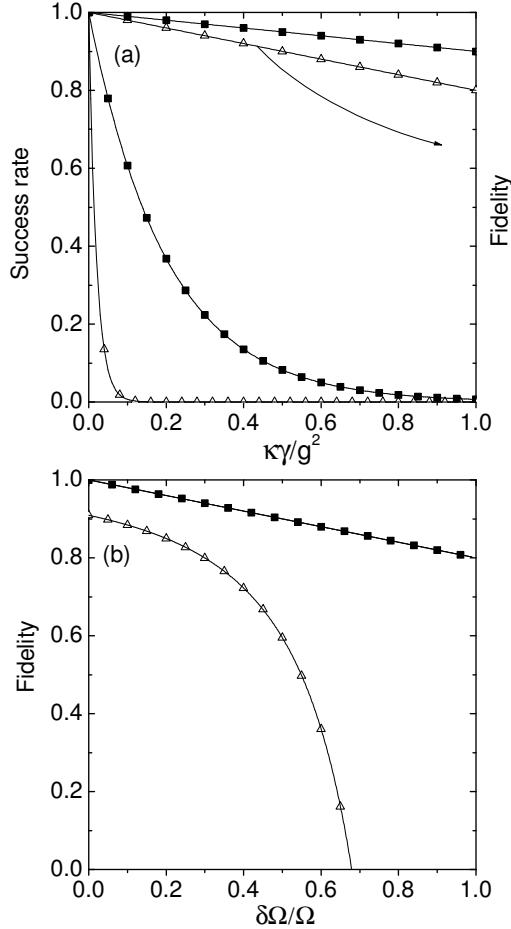


Fig. 3. (a) Success rate and fidelity vs. $\kappa\gamma/g^2$ at the time when $|\Omega_1(t)| = |\Omega_2(t)|$. Solid square denotes the results for this protocol, while open uptriangle corresponds to the results for the setup in Ref.¹⁷. (b) Fidelity vs. $\delta\Omega/\Omega$. Solid square denotes the results for this protocol, and open uptriangle corresponds to the results for the setup in Ref.¹⁹. Other parameters are chosen as in Fig. 2.